The RBC: Calibration & Evaluation

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Macroeconomic Analysis I

The Business Cycle

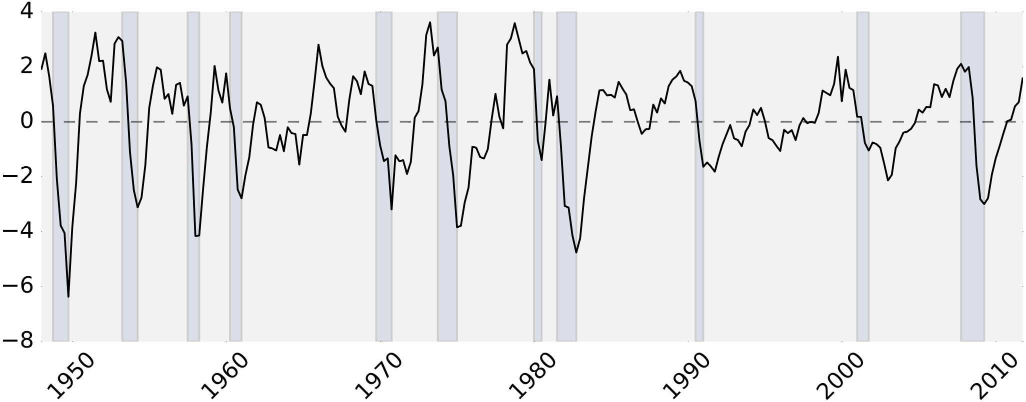

Figure: cyclical component of real GDP (using HP filter)

Recall a few of the basic business cycle facts

- Consumption, investment, and labor are all strongly pro-cyclical
- Labor productivity is moderately procyclical
- Real wages are only mildly pro-cyclical
- The Solow residual (a proxy for TFP) is strongly pro-cyclical

The Strong Comovement of Y, C, and I

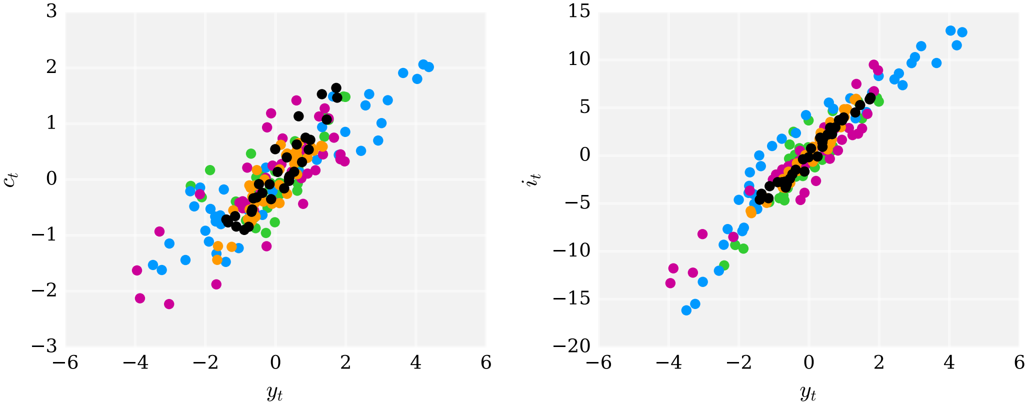

Figure: Scatterplots. Left: output and consumption. Right: output and investment.

Questions

- Does the RBC model help us understand the business cycle fluctuations observed in the real world?
- That is, does the RBC framework give us a complete and coherent structural interpretation of the data?
- Or does the RBC give us a narrative that is:
  - incomplete (i.e. fails to spell out the key assumptions)
  - or even incoherent! (i.e. impossible to reconcile with the data)

What we will do in this lecture

1. Calibration of the RBC

2. Evaluation of the calibrated RBC

Calibration

Calibration

The core idea behind calibration is the following:

1. we parameterize the model (choose functional forms)
2. choose values for parameters so as to match some basic macro and/or micro facts
3. simulate the model to generate artificial data about the business cycle
4. compare the model's artificial data to the actual data.

Parameterization

- Step 1: parameterize the utility function and the production function in the model.
- Following Cooley and Prescott (1995),
  - preferences separable in consumption and leisure

$$U(c,\ell) = (1-b)\log c + b\log(1-\ell), \qquad b \in (0,1)$$

  - can think of total endowment of time as 1, leisure as $1-\ell$
  - Cobb-Douglas technology

$$F(k,\ell) = k^{\alpha}\ell^{1-\alpha}, \qquad \alpha \in (0,1)$$

Why Cobb Douglas technology?

- Cobb-Douglas production function

$$F(k,\ell) = k^{\alpha}\ell^{1-\alpha}$$

$$MPL_t = z_t F_L(k_t,\ell_t) = (1-\alpha)\frac{y_t}{\ell_t} \qquad \text{and} \qquad MPK_t = z_t F_K(k_t,\ell_t) = \alpha\frac{y_t}{k_t}$$

- capital and labor shares of output have been roughly constant over time
    - not exactly true anymore: labor share has been declining

Parameterization

- We next assume that TFP follows a log-normal AR(1) with drift:

$$\log z_{t+1} = (1-\rho)\mu t + \rho \log z_t + \varepsilon_{t+1}, \qquad \varepsilon_{t+1} \sim N(0,\sigma^2)$$

- $\mu > 0$ is the drift (average growth rate)
- $\rho \in (0,1)$ is the auto correlation coefficient (persistence)
- σ is the standard deviation of innovations.

Calibration

- The model is then identified by the following vector of parameters

$$(\beta, b, \alpha, \delta, \mu, \rho, \sigma)$$

- Step 2: specify particular values for these parameters.
- We will follow (approximately) the same calibration strategy as in Cooley and Prescott (1995)

How to choose parameter values

- consider the optimality conditions of the model
- evaluate them at their non-stochastic steady state (or, to be more precise, on the balanced-growth path)
- combine the steady-state conditions with actual US data to solve for the values of the aforementioned parameters that make the model's steady state match the long-run characteristics
- note that we are choosing the model's parameter values to match certain long-run facts, not business-cycle facts!

Equilibrium System of Equations

- With our specifications for the utility and production functions, the system of equations that characterize the equilibrium allocations in the economy are:

$$\frac{b}{1-b}\frac{c_t}{1-\ell_t} = (1-\alpha)\frac{y_t}{\ell_t}$$

$$\frac{1}{c_t} = \beta E_t\left[\left(1-\delta+\alpha\frac{y_{t+1}}{k_{t+1}}\right)\frac{1}{c_{t+1}}\right]$$

$$c_t + k_{t+1} = (1-\delta)k_t + y_t$$

$$y_t = z_t k_t^{\alpha}\ell_t^{1-\alpha}$$

Steady State

- Consider now the "steady state" of the model: the non-stochastic balanced growth path

$$\frac{c_{t+1}}{c_t} = \frac{k_{t+1}}{k_t} = \frac{y_{t+1}}{y_t} = 1 + g$$

  - c_t, k_t, and y_t all grow at a common and constant rate g

- Using the production function we have that

$$\frac{y_{t+1}}{y_t} = \frac{z_{t+1} k_{t+1}^{\alpha} (\ell^*)^{1-\alpha}}{z_t k_t^{\alpha} (\ell^*)^{1-\alpha}} = \left(\frac{z_{t+1}}{z_t}\right)\left(\frac{k_{t+1}}{k_t}\right)^{\alpha}$$

- Therefore

$$(1+g) = (1+\mu)(1+g)^{\alpha}$$

- Solving this for g gives us

$$(1+g) = (1+\mu)^{1/(1-\alpha)}$$

Steady State equations

- in the steady state, these are all constant:

$$\left\{\ell_t, \frac{c_t}{y_t}, \frac{k_t}{y_t}\right\} = \left\{\ell^*, \left(\frac{c}{y}\right)^*, \left(\frac{k}{y}\right)^*\right\}$$

- this implies that the above conditions become:

$$\frac{b}{1-b}\frac{\ell^*}{1-\ell^*} = \frac{(1-\alpha)}{(c/y)^*}$$

$$\frac{c_{t+1}}{c_t} = 1+g = \beta\left(1-\delta+\alpha\,(y/k)^*\right)$$

$$(c/y)^* + (1+g)(k/y)^* = (1-\delta)(k/y)^* + 1$$

- furthermore, under perfect competition, α coincides with the income share of capital:

$$1-\alpha = \left(\frac{w\ell}{y}\right)^*$$

Steady State Values

- From micro studies of time use, households allocate approximately 1/3 of their discretionary time to market activities (discretionary time excludes sleeping), so that

$$\frac{\ell^*}{1-\ell^*} \approx \frac{1/3}{2/3} = \frac{1}{2}$$

- from US macro data (appropriately adjusted as discussed in detail in Cooley and Prescott), we get

$$\begin{aligned} g &\approx .01 \\ \left(\frac{w\ell}{y}\right)^* &\approx .6 \\ (c/y)^* &\approx .75 \\ (k/y)^* &\approx 4\,(3.5) \end{aligned}$$

  - that is, a k/y ratio of 3.5 at annual frequency

Implied parameters from steady state

- we plug in these numbers into the steady-state equations
- solving for the values for the model's parameters, we obtain

$$\mu \approx .006, \qquad \alpha \approx .4, \qquad b \approx .60$$

$$\beta \approx .987 \quad \text{(i.e. discount rate of 5\% per year)}$$
$$\delta \approx .012 \quad \text{(i.e. depreciation of 5\% per year)}$$

- Finally, if we construct the Solow residual, we can estimate its process and get

$$\rho \approx .95, \;\; \sigma \approx .007$$

- We have thus obtained specific values for all the parameters of the model

Evaluation of the RBC

Step 3: Numerical Simulation

- Simulate the model to generate artificial data about the business cycle
- We can generate artificial data from the model
- Compute certain statistics on those artificial data
- And finally compare them to the corresponding statistics in the actual data

Generate an artificial time series

- In particular, consider the following exercise:
- First, use a random-number generator in the computer to generate a particular sequence of TFP shocks.
- Next, use the model to generate the sequence of macroeconomic outcomes that corresponds to this particular sequence of TFP shock.
- In this way you construct a particular artificial time series

Generate many artificial time series

- Repeat the same exercise many many times.
- In this way you will have generated many artificial time series.
- The more such artificial time series you generate, the better you can approximate the theoretical moments of the model by taking averages across these times series.

Compute moments from the model-generated data

- The particular moments that we are interested in are those of the cyclical components of macroeconomic series—not of the entire macro series.
- As mentioned before, the standard practice is to identify the cyclical component by using either the HP or the Bandpass filter.
- The empirical evaluation of the model is then conducted by computing the moments of the filtered artificial data and comparing them to the filtered actual data.
- You may have some freedom to choose the filter, but you'd better apply the same filter on both actual and artificial data

Step 4: Comparison to the data

- Suppose we use the HP filter on the model-generated data
- We then get Tables 1.1 and 1.2 in Cooley and Prescott (1995)
  - Table 1.1 gives various moments of the actual data
  - Table 1.2 gives various moments of the model-generated artificial data
- The key moments of interest are given in the following table

Step 4: Comparison to the data

		st.dev.%	st.dev.%		corr(x,y)	corr(x,y)
		US	**model**		**US**	**model**
output		1.72	1.35		1	1
consumption (non-durables)		0.86	0.33		.77	.84
consumption (total)		1.27	0.33		.83	.84
investment		8.24	5.95		.91	.99
hours (household survey)		1.59	0.77		.86	.99
hours (establishment survey)		1.69	0.77		.92	.99
labor productivity (household survey)		.90	0.61		.41	.98
labor productivity (establishment survey)		.73	0.61		.34	.98

Table 1: The prototype RBC model, from Cooley and Prescott (1995)

Assessment of the RBC's empirical performance

- As evident from this table, our simple model does a pretty good job in matching a number of stylized facts for the US economy.
- We see that that model does a very good job in matching the facts that:
  - consumption, investment, and hours (labor) are all strongly procyclical (columns 3 & 4)
  - investment is highly volatile (relative to the volatility of output)
- but the model somewhat misses on the following facts:
  - the model fails to generate enough volatility in hours
  - the model fails to generate enough volatility in consumption (although it matches the fact that non-durables are less volatile that output)
  - the model also over-predicts the procylicality of labor productivity

Conclusion

- it is actually quite impressive that model can do such a good job in matching the facts
  - the RBC model can match correlations of most variables with output
  - however it has a harder time in matching the observed volatility in consumption and worker-hours
  - it also over-predicts the procyclicality of labor productivity
- this is *despite* the fact that:
  - we calibrated the basic RBC model to long-run growth facts
  - we fed in a time-series of productivity (TFP) shocks
  - we didn't do any calibration of the model to match short-run business-cycle moments

Conclusion

- in conclusion, to answer our previously-posed questions:
- in my opinion, the basic RBC model gives us a coherent, yet incomplete interpretation of the data
- it can match many business cycle facts, but not all facts