Intuition for the RBC

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As we have seen so far, the basic Real Business Cycle (RBC) model is the Neoclassical Growth model with stochastic TFP and endogenous labor supply. In this lecture we take a closer look at the intratemporal and intertemporal (Euler) conditions in order to gain some intuition for how the RBC generates fluctuations in endogenous variables.

1 The Intratemporal Condition

Recall that the household's intratemporal optimality condition is given by:

$$-\frac{U_\ell(s^t)}{U_c(s^t)} = w_t(s^t), \qquad \forall t, s^t.$$

That is, the household sets its marginal rate of substitution between consumption and labor equal to the real wage, i.e. the price ratio between consumption and labor.

Next, recall that the firm's FOCs are given by:

$$r_t(s^t) = z(s^t)F_K(s^t) \quad \text{and} \quad w_t(s^t) = z(s^t)F_L(s^t), \qquad \forall t, s^t \tag{1}$$

The firm's optimality conditions are such that prices are equal to their respective marginal products. Combining this with the household's intratemporal condition, we get:

$$-\frac{U_\ell(s^t)}{U_c(s^t)} = w_t(s^t) = z(s^t)F_L(s^t), \qquad \forall t, s^t.$$

In equilibrium, the marginal rate of substitution between consumption and labor is equal to its marginal rate of transformation, i.e. the marginal product of labor.

1.1 Wealth vs. Substitution Effects

Let's first examine the household's side of this equation.

$$-\frac{U_\ell(s^t)}{U_c(s^t)} = w_t(s^t), \qquad \forall t, s^t.$$

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This equation determines the household's optimal tradeoff between consumption and labor. Let's consider a partial equilibrium analysis for the change in the price ratio.

Assume utility is additively-separable in consumption and labor:

$$U(c,\ell) = u(c) - \phi(\ell) \tag{2}$$

where u, v are both continuous, twice-differentiable functions. We assume $u(\cdot)$ is increasing and concave, $u' > 0$ and $u'' < 0$, and that the function $\phi(\cdot)$ is increasing and convex:

$$\phi' > 0 \qquad \text{and} \qquad \phi'' > 0$$

With additively separable utility, the household's condition can be written as

$$\frac{\phi'(\ell_t(s^t))}{u'(c_t(s^t))} = w_t(s^t), \qquad \forall s^t \in S^t.$$

Suppose for a moment we hold the term $u'(c(s^t))$ constant. What is the effect of an increase in the worker's wage? Because ϕ is increasing and convex, we know that ϕ' is an increasing function of ℓ. Thus, an increase in the wage should increase labor supply. This is the substitution effect.

But in addition to the substitution effect there is also an income or wealth effect which works in the opposite direction. The wage bump *increases* the household's income. Because the household now feels richer, it would like to consume both more consumption (goods) and more leisure. That is, it would like to work less! This is because leisure a normal good.

The wealth effect can be seen in the $u'(c(s^t))$ term. This is the marginal utility of consumption of the household, and by the envelope theorem it is also the marginal utility of wealth:

$$\lambda_t(s^t) = U_c(s^t) = u'(c(s^t)).$$

Therefore, the income (or wealth) effect of an increase in the wage works in the opposite direction of the substitution effect.

1.2 Homothetic Preferences and Cobb-Douglas Technology

We now consider the full general equilibrium. We impose some functional form assumptions; in particular we assume homothetic preferences. Let

$$u(c) \equiv \frac{c^{1-\gamma}}{1-\gamma} \qquad \text{and} \qquad \phi(\ell) \equiv \frac{\ell^{1+\epsilon}}{1+\epsilon}. \tag{3}$$

for $\gamma > 0$ and $\epsilon > 0$. This implies

$$u'(c) = c^{-\gamma} \qquad \text{and} \qquad \phi'(\ell) = \ell^{\epsilon}.$$

Let us also impose Cobb-Douglas Production

$$F(K,L) = K^{\alpha} L^{1-\alpha}$$

Thus

$$F_K(K,L) = \alpha \left(\frac{L}{K}\right)^{1-\alpha} \text{ and } F_L(K,L) = (1-\alpha)\left(\frac{K}{L}\right)^{\alpha}.$$

With homothetic preferences and Cobb-Douglas technology, the equilibrium intratemporal condition is given by:

$$\frac{L_t(s^t)^{\epsilon}}{C_t(s^t)^{-\gamma}} = (1-\alpha) z(s_t) \left(\frac{K_t(s^{t-1})}{L_t(s^t)}\right)^{\alpha}. \tag{4}$$

Again for a moment let's hold the wealth effect $C_t(s^t)^{-\gamma}$ term constant. Consider an increase in productivity. We can see immediately from the right hand side of equation (4) that an increase in z leads to an increase in the *marginal product* of labor. This increases the firm's demand for labor. As a result, for any L employed by the firm, the firm is willing to pay a higher wage. From the left hand side of equation (4), an increase in the wage leads to an increase in the labor supply of the household (holding fixed the wealth effect).

In fact, we can rewrite (4) as follows:

$$\frac{L_t(s^t)^{\epsilon+\alpha}}{C_t(s^t)^{-\gamma}} = (1-\alpha) z(s^t) K_t(s^{t-1})^{\alpha}$$

From here it is clear that an increase in productivity should lead to an increase in equilibrium labor. Holding the denominator $C_t(s^t)^{-\gamma}$ constant, an increase in $z(s_t)$ leads to an increase in the marginal product of labor and hence to a higher level of equilibrium labor, $L_t(s^t)$. One can write this in logs as:

$$\log L_t(s^t) = \frac{1}{\epsilon+\alpha}\left[\log z(s_t) + \alpha \log K_t(s^{t-1})\right] + \cdots$$

Therefore, the sensitivity of equilibrium labor to productivity shocks depends on the parameters

$$\frac{1}{\epsilon+\alpha}$$

Why? First, the parameter ϵ determines the curvature of the household's disutility of labor. We call $1/\epsilon$ the *Frisch elasticity of labor supply.* If the household's disutility of labor is very convex, i.e. ϵ is high, then the household's labor supply is very inelastic. In this case its labor supply won't respond much to a wage increase. On the other hand if the household's disutility of labor is close to linear, i.e. ϵ is low/close to zero, then the household's labor supply is very elastic. In this case its labor supply will respond a lot to an increase in the wage.

The parameter α works in basically the same way but on the firm's side. Recall that the labor share of output is $1-\alpha$. If α is high (close to 1), then there is a lot of diminishing marginal product of labor. Hence an increase in productivity may not move the firm's demand for labor so much. If, on the other hand if α is very low/close to 0, then the firm's technology is almost linear in labor. In this case an increase in productivity would greatly increase the firm's demand for labor.

Finally, note that capital here is isomorphic to productivity. If you enter the period with more capital (which is basically exogenous at that point), it is *as if* you have greater productivity (for any given z), i.e. capital increases the marginal product of labor. In fact, one could have redefined effective productivity as $\hat{z}(s^t) \equiv z(s^t)K_t(s^{t-1})^\alpha$ and all of the same intuition given above would hold, in particular:

$$\log L_t(s^t) = \frac{1}{\epsilon + \alpha}\left[\log \hat{z}(s^t)\right] + \cdots$$

1.3 The Wealth (or Income) Effect

Now consider the wealth (or income) effect embedded in the $C_t(s^t)^{-\gamma}$ term. As already explained above, this has an opposite effect on labor supply.

Clearly the wealth (or income) effect is governed by the parameter γ. If you are using homothetic utility as in (3), to ensure that the wealth effect is small, you would need to set a low γ. But this is only when γ parameterizes the wealth (income) effect *alone*. In most cases it parameterizes other things: risk aversion and the elasticity of intertemporal substitution. In those cases you may want a different value for γ. In other words, one needs to be careful in understanding what certain parameters do in the model.

Finally, if one wants to kill the wealth effect completely, you can use what are known as Greenwood, Hercowitz and Huffman (1988), or GHH, preferences;. These are given by

$$U(c, \ell) = u(c - \phi(\ell))$$

so that utility is not additively separable in consumption and labor. As a result of these preferences, the intratemporal condition looks like the following:

$$\frac{u'(\cdot)\phi'(\ell_t(s^t))}{u'(\cdot)} = w_t(s^t), \qquad \forall t, s^t.$$

which reduces to

$$\phi'(\ell_t(s^t)) = w_t(s^t), \qquad \forall t, s^t.$$

Hence you kill the income effect completely and are left with only the substitution effect on household labor. It's a bit of an extreme way of getting it, but it's good to be able to recognize these tricks.

Why kill the wealth effect? For macroeconomists it appears that in the macro data the substitution effect outweighs the wealth effect: labor (worker-hours) is procyclical, i.e. labor goes up in booms and falls in recessions.

2 The Intertemporal Condition

We now consider the household's intertemporal, or Euler, condition. Recall that the household's intertemporal condition (FOC with respect to capital) is given by

$$U_c(s^t) = \beta\mathbb{E}\left[U_c(s^{t+1})\left(1 + r_{t+1}(s^{t+1}) - \delta\right)|s^t\right], \qquad \forall s^t \in S^t;$$

Combining this condition with the firm's optimality condition for capital (1), we get:

$$U_c(s^t) = \beta \mathbb{E}\left[U_c(s^{t+1})\left(1 + z(s_{t+1})F_K(s^{t+1}) - \delta\right)|s^t\right], \qquad \forall s^t \in S^t;$$

That is, in equilibrium the marginal rate of substitution between consumption today and consumption tomorrow is equal to the marginal rate of transformation (in expectation).

2.1 Steady-state consumption

To gain some intuition, let us first consider the case of no uncertainty. Consider again the case in which utility is additively separable in consumption in labor (2).

In this case we may write the Euler equation as follows

$$u'(C_t) = \beta(1 + r_{t+1} - \delta)u'(C_{t+1}), \qquad \forall t$$

Note that if

$$\beta(1 + r_{t+1} - \delta) = 1, \qquad \forall t,$$

then $u'(C_t) = u'(C_{t+1})$, which implies that

$$C_t = C_{t+1}, \qquad \forall t.$$

We call this a flat consumption path: consumption is not growing. This occurs when the discount rate is equal to the interest rate (rental rate minus depreciation):

$$\rho \equiv \frac{1}{\beta} - 1 = r - \delta$$

where ρ is what is called the discount *rate* (as opposed to the discount factor β).

If the household discounts at the same rate as the interest rate, consumption is equal across all periods. We can think of this as the steady state of the model.

2.2 An "unexpected shock" to productivity

Suppose we are in steady state, so that $\beta(1 + r_{t+1} - \delta) = 1$ and

$$C_t = C_{t+1}, \qquad \forall t.$$

Starting from the steady-state with no uncertainty, we allow for what is called an "MIT" shock.

Suppose there is an unexpected, persistent, positive shock to productivity at time t. What happens to the intertemporal condition? First consider the rental rate on capital next period, given by:

$$r_{t+1}(s^{t+1}) = z(s^{t+1})F_K(s^{t+1})$$

Because the shock is persistent, the marginal product of capital is higher than originally expected. Thus

$$\beta(1 + r_{t+1} - \delta) > 1.$$

The Euler equation implies

$$u'(C_t) > u'(C_{t+1})$$

and because U is a concave function, this implies that

$$C_t < C_{t+1}$$

Therefore, the household's consumption path is increasing. A positive, persistent productivity shock means that the future marginal product of capital is higher which translates into a higher rental rate on capital, i.e. a higher return on capital. A greater return on capital incentivizes the household to invest more in physical capital, which is why we should expect the consumption path to grow.

Consider now the opposite MIT shock: an unexpected, persistent, negative shock to productivity at time t. What happens to the intertemporal condition? In this case the marginal product of capital is lower than originally expected. As a result,

$$\beta(1 + r_{t+1} - \delta) < 1,$$

The Euler equation implies

$$u'(C_t) < u'(C_{t+1})$$

and because u is a concave function, this further implies that

$$C_t > C_{t+1}$$

Therefore, the household's consumption path is decreasing. A negative, persistent productivity shock means that the future marginal product of capital is lower which translates into a lower return on capital. A lower return on capital incentivizes the household to invest less in physical capital, which is why we should expect the consumption path to fall.

Therefore, whether an individual's consumption increases or decreases over time depends on the interest rate. All else equal, a higher interest rate encourages greater savings and investment. How much the household saves (invests) depends on how high the interest rate is relative to the discount rate, ρ.

2.3 The Elasticity of Intertemporal Substitution

Let's now again assume that utility is separable in consumption in labor (2) and that preferences are homothetic (3). Let $\theta \equiv 1/\gamma$; thus the flow utility from consumption can be written as:

$$u(c) \equiv \frac{c^{1-1/\theta}}{1-1/\theta}$$

where θ is the elasticity of intertemporal substitution (EIS). In this case our Euler equation may be written as follows

$$C_t^{-1/\theta} = \beta(1 + r_{t+1} - \delta)C_{t+1}^{-1/\theta}, \quad \forall t,$$

or alternatively

$$\left(\frac{C_{t+1}}{C_t}\right)^{1/\theta} = \beta(1 + r_{t+1} - \delta), \quad \forall t.$$

We can rewrite this as

$$\left(\frac{C_{t+1}}{C_t}\right) = [\beta(1 + r_{t+1} - \delta)]^{\theta}, \quad \forall t. \tag{5}$$

Finally, taking logs of both sides we get:

$$\log C_{t+1} - \log C_t = \theta \log\{\beta(1 + r_{t+1} - \delta)\} \tag{6}$$

If $\beta(1 + r_1 - \delta) = 1$, then the right hand side is equal to zero, and the household equalizes consumption over periods no matter what his or her utility function is.

Now suppose $\beta(1 + r_{t+1} - \delta) > 1$. Then as we showed previously, consumption will increase over time: $\log C_{t+1} - \log C_t > 0$. One can think of $\log C_{t+1} - \log C_t$ as the percentage change in consumption. The question we ask now is: how *sensitive* is this change in consumption with respect to changes in the interest rate?

The answer to this question depends on the value of θ. Think of it this way. The household faces a trade-off: a higher interest rate encourages savings and therefore encourages consumption growth. On the other hand, the household would like consumption to be smooth over time, so it may prefer to not save so much in order to have a flatter consumption path. What determines the sensitivity of the change in consumption to the interest rate is the EIS θ.

If the elasticity of intertemporal substitution θ is high, then the percentage change in consumption, $\log C_{t+1} - \log C_t$, is *very* responsive to movements in $\beta(1 + r_{t+1} - \delta)$. In this case, the household does not have a very strong consumption smoothing motive—utility is almost linear—and hence it would like to take major advantage of a high interest rate. Thus, the household would save a lot in this period in order to consume more in the future.

If on the other hand the elasticity of intertemporal substitution θ is very low (close to zero), then the utility function has a lot of curvature—the household is inelastic in terms of intertemporal substitution. In this case the percentage change in consumption, $\log C_{t+1} - \log C_t$, is *not very* responsive to changes in $\beta(1+r_{t+1}-\delta)$. The household has a high consumption smoothing motive: more curvature in the utility function implies the household would prefer to have a flat consumption path. A higher return on capital would thus have a substantially smaller effect in encouraging the household to depart from a flat consumption path.

We may rewrite (6) as follows

$$\log C_{t+1} - \log C_t = \theta\{\log(1 + r_{t+1} - \delta) - \log(1/\beta)\} = \theta\{\log(1 + r_{t+1} - \delta) - \log(1 + \rho)\}$$

If $r_{t+1} - \delta$ and ρ are small, we obtain the following approximation

$$\log C_{t+1} - \log C_t \approx \theta\{r_{t+1} - \delta - \rho\}$$

Therefore consumption is growing if and only if $r_{t+1} - \delta - \rho > 0$. The *elasticity of intertemporal substitution* with respect to a change in the interest rate is $\theta = 1/\gamma$.

3 Summary

Hopefully this lecture provides a better understanding and intuition for how the endogenous variables in our models should move in response to TFP shocks. Although I explained the intuition using unexpected "MIT" shocks, the intuition is the same even in the full model in which agents have Rational Expectations, i.e. they understand the underlying shock structure of the economy and expect shocks to occur.

The main idea is that a positive shock to TFP at time t leads to an increase in both the marginal product of labor and the marginal product of capital. On impact, labor endogenously increases as long as substitution effects are sufficiently strong. Capital on impact is not affected because it is taken as given once you enter the period. Thus current output should go up because both z_t and L_t increase.

If the increase in z_t is persistent, then it also increases the marginal product of capital in following periods. This induces the household to save more today, therefore invest more today, which implies greater capital in the following periods. As a result, the marginal product of labor in the next period would be higher not only due to the increase in productivity but also due to the greater capital stock. (Recall that the effects of TFP and capital on labor are isomorphic.) As a result, labor and output would increase the next period as well, and so on.

References

Greenwood, Jeremy, Zvi Hercowitz, and Gregory W. Huffman, "Investment, Capacity Utilization, and the Real Business Cycle," *The American Economic Review*, 1988, *78* (3), 402–417.